(7 pages) **Reg. No. :**..

Code No. : 20573 E **Sub. Code : SMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Fourth Semester

Mathematics — Core

ABSTRACT ALGEBRA — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The order of the element 3 in $(Z_8, \oplus)$ is ___________.

   (a) 2 (b) 4

   (c) 6 (d) 8

2. Which one of the following is a group?

   (a) $(Z_6, \odot)$ (b) $(Z_7, \odot)$

   (c) $(Z_{11} - \{0\}, \odot)$ (d) $(Z_8 - \{0\}, \odot)$

3. The set of generators of the group $(Z_{12}, \oplus)$ is __________.

(a) $\{1, 2, 3, 4\}$ (b) $\{1, 3, 6, 9\}$

(c) $\{1, 5, 7, 11\}$ (d) $\{2, 3, 5, 7\}$

4. Number of elements in the cyclic subgroup $\langle 2 \rangle$ of $(Z_{18}, \oplus)$ is

(a) 1 (b) 18

(c) 9 (d) 5

5. The number of elements in the quotient group $Z_{60}/\langle 5 \rangle$ is

(a) 3 (b) 5

(c) 15 (d) 20

6. If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ is a permutation then α^{-1} is __________.

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

7. Which one of the following is not a ring?

(a) $(\mathbb{Z}, +, \cdot)$ (b) $(\mathbb{Q}, +, \cdot)$

(c) $(Z_n, \odot, \oplus)$ (d) $(Z_n, \oplus, \odot)$

8. In the ring $\mathbb{Z}$, (n) is a maximal ideal $\Leftrightarrow$ __________.

(a) n is a prime number

(b) n is a composite number

(c) $n \neq 2$

(d) $n > 13$

9. Let $f : \mathrm{C} \to \mathbb{C}$ be defined by $f(z) = \bar{z}$. Then kerf is __________.

(a) φ (b) $\{0\}$

(c) $\{1\}$ (d) $\{i\}$

10. Field of quotients of is $\mathbb{Q}$ is __________

(a) Q (b) N

(c) $\mathbb{Z}$ (d) None

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If G is a finite group with even number of elements then prove that G contains atleast one element of order 2.

Or

(b) If H and K are subgroups of a group G then prove that $H \cap K$ is also a subgroup of G.

12. (a) Define a cyclic group. Prove that a subgroup of a cyclic group is cyclic.

Or

(b) State and prove Euler's theorem.

13. (a) Prove that any infinite cyclic group is isomorphic to $(\mathbb{Z}, +)$.

Or

(b) Let N be a normal subgroup of a group G. Show that G/N is a group.

14. (a) Let R be a ring and $a, b \in R$. Prove that

(i) $0.a = a.0 = 0$

(ii) $a(-b) = (-a)b = -(ab)$

(iii) $(-a)(-b) = ab$

(iv) $a.(b-c) = a.b - a.c$.

Or

(b) Let R be a commutative ring with identity and P be an ideal of R. Prove that P is a prime ideal iff R/P is an integral domain.

15. (a) Let R and R' be rings and $f : R \to R'$ be a homomorphism. Prove that $\ker f = \{0\}$ iff f is one-one.

Or

(b) Prove that $R[x]$ is an integral domain $\Leftrightarrow R$ is an integral domain.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If 'a' and 'b' are elements of a group G then prove the following :

(i) Order of a = Order of a^{+}

(ii) Order of a = Order of $b^{-1}ab$

(iii) Order of ab = Order of ba

Or

(b) Show that the union of two subgroups of a group G is a subgroup iff one is contained in the other.

17. (a) Prove that the collection of all left cosets forms a partition of the group.

Or

(b) Let H and K be two subgroups of G of finite index in G. Prove that $H \cap K$ is a subgroup of finite index in G.

18. (a) If a group G has exactly one subgroup H of given order then show that H is a normal subgroup of G.

Or

(b) State and prove Cayley's theorem.

19. (a) Let R be a commutative ring with identity. Prove that an ideal M of R is maximal iff R/M is a field.

Or

(b) Prove the following :

(i) Z_n is a field $\Leftrightarrow n$ is prime.

(ii) The characteristic of an integral domain is either 0 or a prime number.

20. (a) State and prove the fundamental theorem of homomorphism for rings.

Or

(b) Prove that every integral domain can be embedded in a field.